

Measurement and Evaluation of the Thermal Diffusivity of Two-Layered Materials

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Received July 1, 1991

Transient methods, such as those with pulse- or stepwise heating, have often been used to measure thermal diffusivity of various materials including layered composite materials. The aim of the present study is to investigate effects of various parameters on the measurement of thermal diffusivity when the transient methods are applied. Mainly a two-layered material in the pulsewise heating method is considered because of its simplicity and usefulness in identifying and determining the effects of the parameters. First, it has been shown that there exists a special condition for determining the thermal diffusivity of a component in the two-layered material whose other relevant thermophysical properties are known. Second, it has been shown that the thickness of the laser-beam absorption layer, which inevitably makes sample material into the two-layered material, may cause a relatively large error when the thermal diffusivity of the base material is high. Finally, it has been derived a definite relation between the apparent thermal diffusivity obtained from the temperature response and the mean thermal diffusivity, which has a physical meaning related to the thermal resistance.

KEY WORDS: pulsewise heating method; temperature response; thermal diffusivity; two-layered material.

1. INTRODUCTION

Transient methods, such as those with pulse- or stepwise heating, have often been used to measure thermal diffusivity of pure metals, alloys, ceramics, etc. These methods have also been used to determine the thermal diffusivity of a component in the two- or three-layered composite materials, using analytical solutions for the two- or three-layered materials in the

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pulsewise heating method [1], and with that for the three-layered material in the stepwise heating method [2, 3]. Although analytical solutions exist for these transient methods, appropriate conditions for the measurement of the thermal diffusivity have not been discussed thoroughly.

In such transient methods, it is often required for sample materials to have laser-beam absorption layer, such as carbon coating layer. Since these samples inevitably become two-layered materials, the thermal diffusivities obtained from the temperature response are merely apparent and sometimes are subject to great errors. However, error caused by the laser-beam absorption layer has not been fully discussed in the literature.

Moreover, needs have arisen for the transient methods to evaluate thermophysical properties of layered composite materials, such as electronic materials and materials resistant to wear, corrosion, and heat. Although resulting data for the temperature response in the transient methods contain information related to the thermophysical properties of the individual layers, the thermal diffusivity obtained from the temperature response at the rear surface is merely apparent and different from the mean thermal diffusivity, which is calculated from the thermal resistance. Then, a definite relation between the apparent and the mean thermal diffusivities is required for the direct evaluation of the thermophysical properties of layered materials in the transient methods.

The objective of the present study is to shed more light on these measurement methods for layered materials, with changing parameters, such as Fourier numbers, ratio of heat capacities, ratio of thermal diffusion times, etc., over wide ranges. A two-layered material in the pulsewise heating method is considered here because of its simplicity and usefulness in identifying and determining the effects of the parameters. First, we determine applicable ranges for various parameters when we measure the thermal diffusivity of one layer in the two-layered material whose other relevant thermophysical properties are known. Second, we evaluate errors which are caused by the existence of the absorption layer on the base materials. Finally, we derive a definite relation between the apparent thermal diffusivity obtained from the temperature response and the mean thermal diffusivity by obtaining an approximate expression for the apparent thermal diffusivity analytically.

2. FORMULATION

An evaluation of the thermal diffusivity of a two-layered material (Fig. 1) has been conducted. In the following, we restrict ourselves to the pulsewise heating method because nearly the same results can be obtained for the stepwise heating method. In the usual pulsewise heating method, the

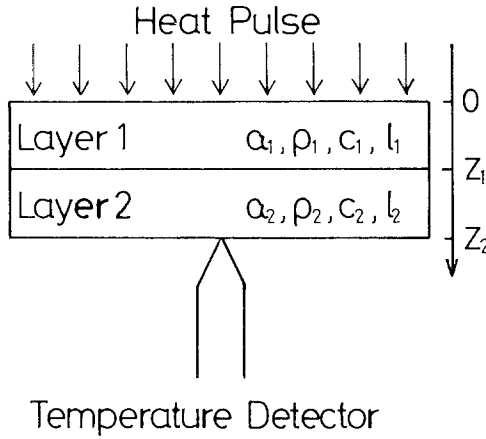


Fig. 1. Schematic diagram of a two-layered material.

front surface is subjected to a short pulse and temperature history at the rear surface is recorded to determine the thermal diffusivity of a sample material; the resulting data for the temperature response contain information related to the thermophysical properties of the individual layers in the sample material. Since the formulation of the present model has already been conducted by Lee [1], herein only the assumptions and the final solution are presented. Solving the heat diffusion equation with the appropriate boundary conditions under the assumptions of

- (1) one-dimensional heat flow,
- (2) no heat loss from the sample surface,
- (3) no thermal contact resistance between layers,
- (4) heat pulse being uniformly absorbed on the front surface,
- (5) each layer being homogeneous, and
- (6) constant thermophysical properties of each layer,

the normalized temperature rise at the rear surface has been derived as

$$V = 1 + 2 \sum_{k=1}^{\infty} \frac{(\omega_1 \chi_1 + \omega_2 \chi_2) e^{-(\gamma_k/\eta_2)^2 t}}{\omega_1 \chi_1 \cos(\gamma_k \omega_1) + \omega_2 \chi_2 \cos(\gamma_k \omega_2)} \quad (1)$$

where

$$\chi_1 = A_{1/2} + 1, \quad \chi_2 = A_{1/2} - 1, \quad \omega_1 = \eta_{1/2} + 1, \quad \omega_2 = \eta_{1/2} - 1 \quad (2)$$

$$A_i = \lambda_i / \sqrt{\alpha_i}, \quad \eta_i = l_i / \sqrt{\alpha_i} \quad (3)$$

$$A_{ij} = A_i / A_j, \quad \eta_{ij} = \eta_i / \eta_j \quad (i, j = 1, 2) \quad (4)$$

and γ_k is the k th positive root of the characteristic equation of

$$\chi_1 \sin(\gamma\omega_1) + \chi_2 \sin(\gamma\omega_2) = 0 \quad (5)$$

Here λ is the thermal conductivity, a the thermal diffusivity, l the length of the layer, A the heat-penetration coefficient, and η the thermal diffusion time. In the work of Lee [1], the following parameters were used:

$$H_i = A_i \eta_i, \quad H_{ij} = H_i / H_j \quad (i, j = 1, 2) \quad (6)$$

If we use $t_{1/2}$, which is the time duration until the temperature rise, V , reaches 50% of its maximum rise, the Fourier number of each layer is expressed as

$$Fo_i = a_i t_{1/2} / l_i^2 = t_{1/2} / \eta_i^2 \quad (i = 1, 2) \quad (7)$$

3. PROBLEMS IN DETERMINING THERMAL DIFFUSIVITY OF MATERIALS

3.1. Determination of Thermal Diffusivity Using a Two-Layered Sample Material

The thermal diffusivities of various materials have been determined using the two-layered materials whose relevant thermophysical properties except for the thermal diffusivity of one layer are known. Although this measurement method is widely used, appropriate selection of materials to be combined for the accurate measurement of the thermal diffusivity has not been fully discussed. In this section, we investigate relations between parameters which influence the determination of the unknown thermal diffusivity.

First, let us investigate the relation between the Fourier numbers of individual layers defined by Eq. (7). Before numerical calculation, we examine several limiting solutions which we can derive analytically. These limiting solutions enable us to confirm the numerical results.

Case I. When $\eta_{1/2} \rightarrow 0$ and $A_{1/2} \rightarrow 0$, that is, $\chi_1 = -\chi_2 = 1$ and $\omega_1 = -\omega_2 = 1$, we have the characteristic equation and its roots as

$$\sin(\gamma) = 0; \quad \gamma = k\pi \quad (k = 1, 2, \dots) \quad (8)$$

From Eq. (1) the temperature rise is expressed as

$$V = 1 + 2 \sum_{k=1}^{\infty} (-1)^k e^{-(k\pi)^2 Fo_2} \quad (9)$$

which yields $Fo_2 = -ln(0.2542)/\pi^2 = 0.1388$ for $V = 0.5$. Since $Fo_2/Fo_1 = \eta_{1/2} = 0$, Fo_1 corresponding to this situation is infinity.

Case II. When $\eta_{1/2} \rightarrow 0$ and $H_{1/2} = A_{1/2}\eta_{1/2} \rightarrow \infty$, we have the characteristic equation and its roots as

$$\tan(\gamma) = 0; \quad \gamma = (2k - 1) \pi/2 \quad (k = 1, 2, \dots) \tag{10}$$

From Eq. (1) the temperature rise is expressed as

$$V = 1 - \left(\frac{4}{\pi}\right) \sum_{k=1}^{\infty} \frac{e^{-[(2k-1)\pi/2]^2 Fo_2}}{(-1)^{k+1} (2k-1)} \tag{11}$$

which yields $Fo_2 = -4ln(0.3927)/\pi^2 = 0.3788$ for $V = 0.5$. Since $Fo_2/Fo_1 = \eta_{1/2} = 0$, Fo_1 corresponding to this situation is infinity.

Case III. When $\eta_{1/2} = 1$ and $A_{1/2} = 1$, that is, $\chi_1 = 2$, $\chi_2 = 0$, $\omega_1 = 2$, and $\omega_2 = 0$, we have the characteristic equation and its roots as

$$\sin(2\gamma) = 0; \quad 2\gamma = k\pi \quad (k = 1, 2, \dots) \tag{12}$$

From Eq. (1) the temperature rise is expressed as

$$V = 1 + 2 \sum_{k=1}^{\infty} (-1)^k e^{-(k\pi/2)^2 Fo_2} \tag{13}$$

which yields $Fo_2 = -4ln(0.2542)/\pi^2 = 0.5551$ for $V = 0.5$. Since $Fo_2/Fo_1 = \eta_{1/2} = 1$, Fo_1 corresponding to this situation is also 0.5551.

Case IV. When $\eta_{1/2} \rightarrow \infty$ and $H_{1/2} = A_{1/2}\eta_{1/2} \rightarrow \infty$, that is, $A_{2/1}\chi_1 = A_{2/1}\chi_2 = 1$ and $\eta_{2/1}\omega_1 = \eta_{2/1}\omega_2 = 1$, we have the characteristic equation and its roots as

$$\sin(\eta_{1/2}\gamma) = 0; \quad \eta_{1/2}\gamma = k\pi \quad (k = 1, 2, \dots) \tag{14}$$

From Eq. (1) the temperature rise is expressed as

$$V = 1 + 2 \sum_{k=1}^{\infty} (-1)^k e^{-(k\pi)^2 Fo_1} \tag{15}$$

which yields $Fo_1 = -ln(0.2542)/\pi^2 = 0.1388$ for $V = 0.5$. Since $Fo_2/Fo_1 = \eta_{1/2} \rightarrow \infty$, Fo_2 corresponding to this situation is infinity.

Case V. When $\eta_{1/2} \rightarrow \infty$ and $H_{1/2} = A_{1/2}\eta_{1/2} \rightarrow 0$, we have the characteristic equation and its roots as

$$\tan(\eta_{1/2}\gamma) = 0; \quad \eta_{1/2}\gamma = (2k - 1) \pi/2 \quad (k = 1, 2, \dots) \tag{16}$$

From Eq. (1) the temperature rise is expressed as

$$V = 1 - \left(\frac{4}{\pi}\right) \sum_{k=1}^{\infty} \frac{e^{-[(2k-1)\pi/2]^2 Fo_1}}{(-1)^{k+1} (2k-1)} \tag{17}$$

which yields $Fo_1 = -4\ln(0.3927)/\pi^2 = 0.3788$ for $V = 0.5$. Since $Fo_2/Fo_1 = \eta_{1/2} \rightarrow \infty$, Fo_2 corresponding to this situation is infinity.

Relation between Fourier numbers for finite values are obtained from numerical calculations. Figure 2 shows the relation between Fourier numbers Fo_1 and Fo_2 , with the ratio of the heat capacities $H_{1/2}$ and the ratio of the thermal diffusion times $\eta_{1/2}$ taken as parameters. It is seen that initially Fo_2 decreases rapidly and then slowly with increasing Fo_1 , depending on both $H_{1/2}$ and $\eta_{1/2}$. We see that the Fourier numbers are determined uniquely as $Fo_1 = Fo_2 = 0.5551$, regardless of $H_{1/2}$, when $\eta_{1/2} = 1$, as is pointed out for *Case III*, point 4 in Fig. 2. This means that we can determine Fo_2 (or Fo_1) from Fo_1 (or Fo_2) at this point even though the ratio of the heat capacities of the two layers are unknown. Therefore, it is recommended to choose this condition for the measurement of the thermal diffusivity with two-layered material. It is also noted that Fo_2 (or Fo_1) changes drastically when Fo_1 (or Fo_2) is small. In this range,

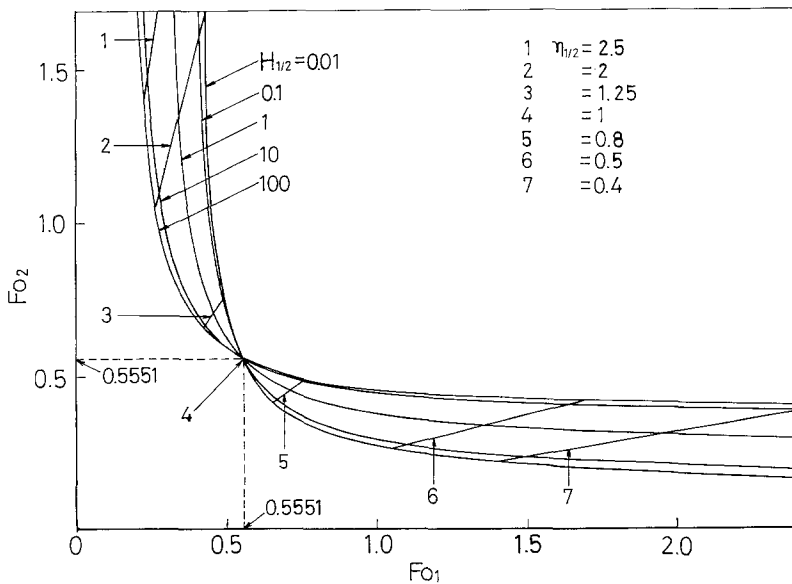


Fig. 2. Relation between the Fourier number based on the first layer and that based on the second layer, with the ratio of the heat capacities and the ratio of the thermal diffusion times taken as parameters.

the determination of Fo_2 (or Fo_1) from given Fo_1 (or Fo_2) may inevitably have a great error because given that Fo_1 (or Fo_2) itself has uncertainty in practical situations.

Figure 3 shows the relation between Fo_1 and $H_{2/1}$, with $\eta_{2/1}$ and Fo_2 taken as parameters. When $\eta_{2/1} = 1$ (or $Fo_2 = 0.5551$), Fo_1 is determined as 0.5551 regardless of $H_{2/1}$. When $\eta_{2/1} < 1$ (or $Fo_2 > 0.5551$), Fo_1 decreases with decreasing $H_{2/1}$; when $\eta_{2/1} > 1$ (or $Fo_2 < 0.5551$), Fo_1 increases first slowly and then rapidly with decreasing $H_{2/1}$. We see from Fig. 3 that determination of Fo_1 may have a great error when $H_{2/1}$ is small; that is, the heat capacities of relevant layers are very much different from each other.

3.2. Measurement of Thermal Diffusivity of Carbon Layer Using a Two-Layered Sample Material

In order to confirm the results in the previous section, an experiment has been conducted to measure the thermal diffusivity of the carbon layer which is formed by the spray coating on the copper disk of 10 mm in diameter. In the experiment, the front surface of the copper is subjected to the heat pulse. Since we have obtained the approximate value of the thermal diffusivity of the porous carbon layer from the preliminary experiment, we have prepared three sample materials which have $\eta_{1/2} \approx 1$ depending on the thickness of the copper disk, as listed in Table I. In Table I subscripts 1 and 2 designate copper and carbon, respectively. The thermophysical properties of copper listed in Table II [4] are also used.

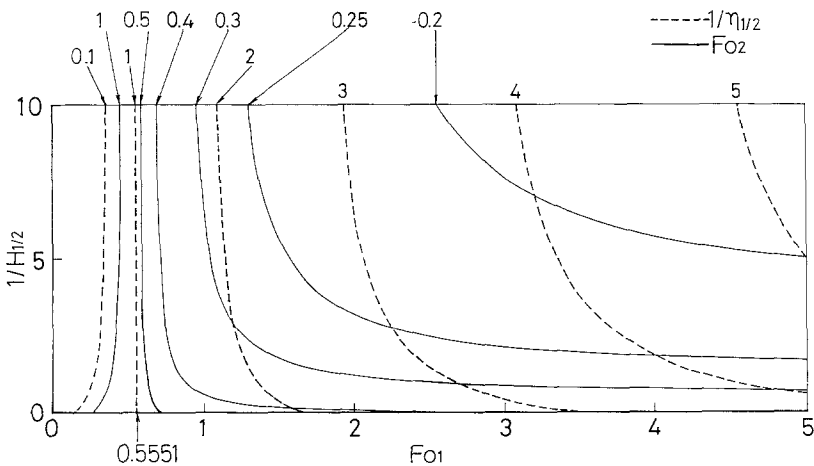


Fig. 3. Relation between Fo_1 and $H_{2/1}$, with Fo_2 and $\eta_{2/1}$ taken as parameters.

Table I. Experimental Results on the Measurement of Thermal Diffusivity of the Carbon Layer Using a Two-Layered Sample Material

	Sample material 1		Sample material 2		Sample material 3	
l_1 (mm)	2.892		2.644		2.002	
l_2 (mm)	0.248		0.190		0.150	
l_1/l_2	11.68		13.95		13.31	
$t_{1/2}$ (ms)	37.32		30.68		18.16	
	A^a	B^b	A^a	B^b	A^a	B^b
ρ_2 ($\text{kg} \cdot \text{m}^{-3}$)	2.20×10^3	0.78×10^3	2.20×10^3	0.95×10^3	2.20×10^3	0.92×10^3
$H_{1/2}$	26.18	74.07	31.25	72.72	29.85	71.42
$\eta_{1/2}$	1.041	1.040	1.054	1.054	1.029	1.029
Fo_1	0.5265	0.5265	0.5179	0.5179	0.5345	0.5345
Fo_2	0.5706	0.5700	0.5756	0.5750	0.5660	0.5657
α_2 ($\text{m}^2 \cdot \text{s}^{-1}$)	9.37×10^{-7}	9.36×10^{-7}	6.74×10^{-7}	6.74×10^{-7}	7.05×10^{-7}	7.05×10^{-7}

^a Density of graphite listed in Table II is used for the determination of the thermal diffusivity of the carbon layer.

^b Measured density of the carbon layer is used for the determination of the thermal diffusivity of the carbon layer.

In Table I, thicknesses of the individual layers, half-times measured at the rear surface of the sample materials, and densities of the carbon layers are listed. Also listed are the values of the parameters, which are determined in the iterative data processing, and the thermal diffusivities of the carbon layers. In column A the density of graphite listed in Table II is used, while in column B the measured density of the carbon layer is used. Since the condition of $\eta_{1/2} \approx 1$ is satisfied, the thermal diffusivity of the carbon layer does not depend on the choice of the density. Although the measured

Table II. Thermophysical Properties of Selected Materials [4]

Material	Density ρ ($\text{kg} \cdot \text{m}^{-3}$)	Specific heat capacity c ($\text{J} \cdot \text{kg}^{-1} \cdot \text{K}^{-1}$)	Thermal diffusivity a ($\text{m}^2 \cdot \text{s}^{-1}$)
Copper	8.94×10^3	3.81×10^2	1.18×10^{-4}
Nickel	8.91×10^3	4.39×10^2	2.28×10^{-5}
Titanium	4.54×10^3	5.27×10^2	7.22×10^{-6}
Quartz	2.21×10^3	7.11×10^2	8.61×10^{-7}
Graphite	2.20×10^3	6.91×10^2	1.57×10^{-5}

thermal diffusivities of carbon layers have different values for the sample materials, we can attribute it to the difference in the densities (or porosities) of the carbon layers.

4. EFFECT OF CARBON COATING LAYER ON THE MEASUREMENT OF THERMAL DIFFUSIVITY

4.1. Apparent Thermal Diffusivity

In the pulswise heating method it is often required that sample materials have absorption layers of the laser beam, such as carbon coating layers. Inevitably, these samples become two-layered composite materials and the thermal diffusivity obtained from the temperature response becomes an apparent one. However, error caused by the coating layer has not been fully discussed in the literature. Here we examine the dependence

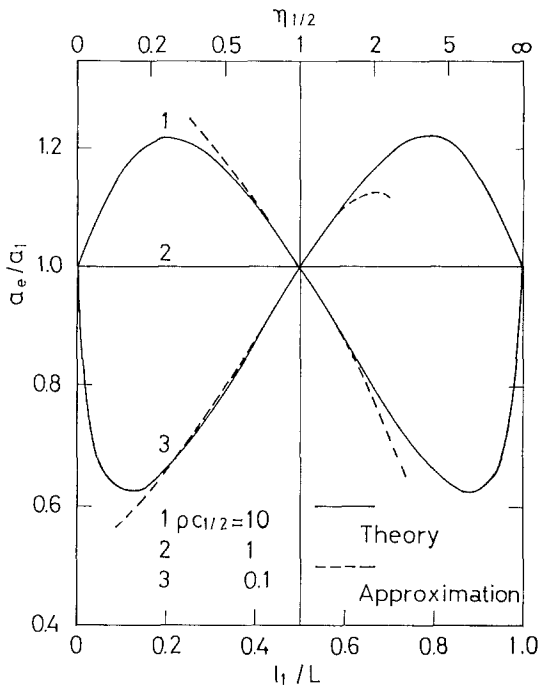


Fig. 4. Apparent thermal diffusivity as a function of the first layer thickness, with the product of the heat capacity and the density taken as a parameter for $a_{1/2} = 1$. Solid curves are obtained from the analytical solution; dashed curves are obtained from the approximate expression.

of the apparent thermal diffusivity of a two-layered material on the thermophysical properties and thickness of individual layers.

The apparent thermal diffusivity a_e of a two-layered material which is regarded as homogeneous can be formally expressed as

$$0.1388 = a_e t_{1/2} / L^2, \quad L = l_1 + l_2 \tag{18}$$

As shown in Figs. 4-7, the apparent thermal diffusivity a_e depends on the ratio of the thermal diffusivities, $a_{1/2}$, the ratio of the heat capacities, $(\rho c)_{1/2}$, and the ratio of the thicknesses, l_1/L . We see in Fig. 4 that the apparent thermal diffusivity a_e changes even when $a_1 = a_2$. This is due to the change of the temperature rise which depends on the heat capacities of

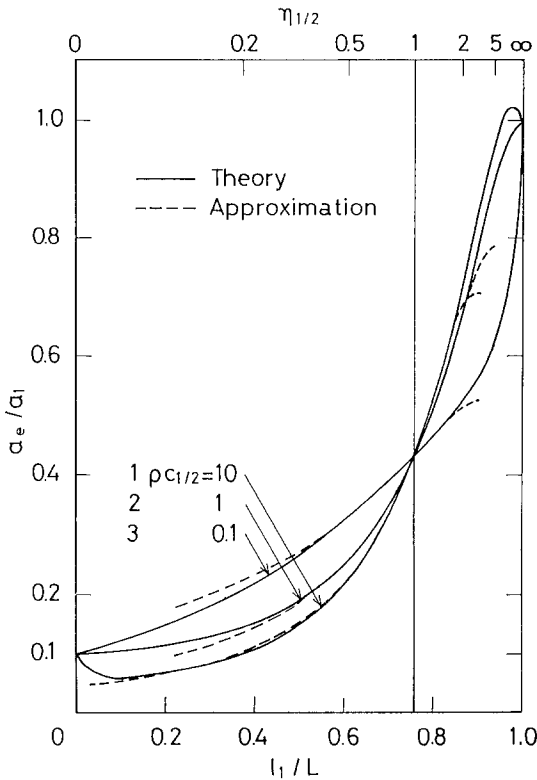


Fig. 5. Apparent thermal diffusivity as a function of the first layer thickness, with the product of the heat capacity and the density taken as a parameter for $a_{1/2} = 10$. Solid curves are obtained from the analytical solution; dashed curves are obtained from the approximate expression.

the layers. The points at which a_e can be determined regardless of the heat capacities satisfy the condition of $\eta_{1/2} = 1$, as pointed out in Section 3.1; $l_1/L = 0.5$ in Fig. 4, $l_1/L = 0.760$ in Fig. 5, $l_1/L = 0.909$ in Fig. 6, and $l_1/L = 0.969$ in Fig. 7.

Although results for the stepwise heating method are not shown here, nearly the same trend is observed as that for the pulsewise heating method shown not only in Figs. 4–7 but also in Figs. 2 and 3. However, it is confirmed that the apparent thermal diffusivity depends on the measurement method, which means that the apparent thermal diffusivity cannot be a thermophysical property and that it must be distinguished from the mean thermal diffusivity which has a physical meaning.

4.2. Errors Caused by the Carbon Coating Layer

As an example, let us examine errors caused by the carbon coating layers which are formed on base materials (Table II) [4] as laser-absorption

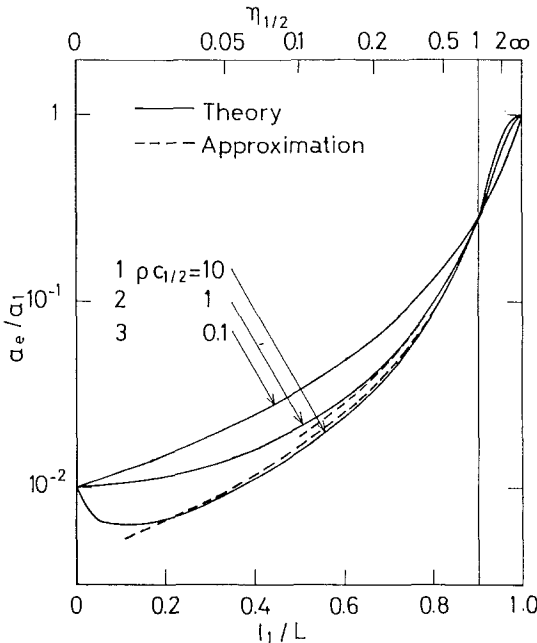


Fig. 6. Apparent thermal diffusivity as a function of the first layer thickness, with the product of the heat capacity and the density taken as a parameter for $a_{1/2} = 100$. Solid curves are obtained from the analytical solution; dashed curves are obtained from the approximate expression.

layers. Figure 8 shows the difference between the Fourier number Fo_2 for the base material and the apparent Fourier number Fo_e defined in Eq. (18), as a function of the thickness ratio of the carbon layer to the base material layer. We see that the difference, that is, error increases with increasing thickness ratio. It is also shown that the error becomes large when the thermal diffusivity of the base material is high. When we consider the situation that there exists carbon coating layer ($l_1 = 30 \mu\text{m}$) on copper sample ($l_2 = 3 \text{ mm}$), the apparent thermal diffusivity is 7% less than the true thermal diffusivity. We note that the thickness of the carbon coating layer may cause a relatively large error for materials of high thermal diffusivities.

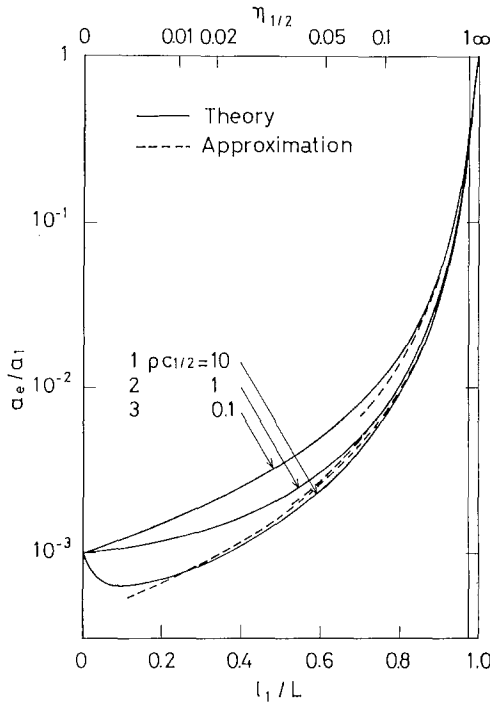


Fig. 7. Apparent thermal diffusivity as a function of the first layer thickness, with the product of the heat capacity and the density taken as a parameter for $a_{1/2} = 1000$. Solid curves are obtained from the analytical solution; dashed curves are obtained from the approximate expression.

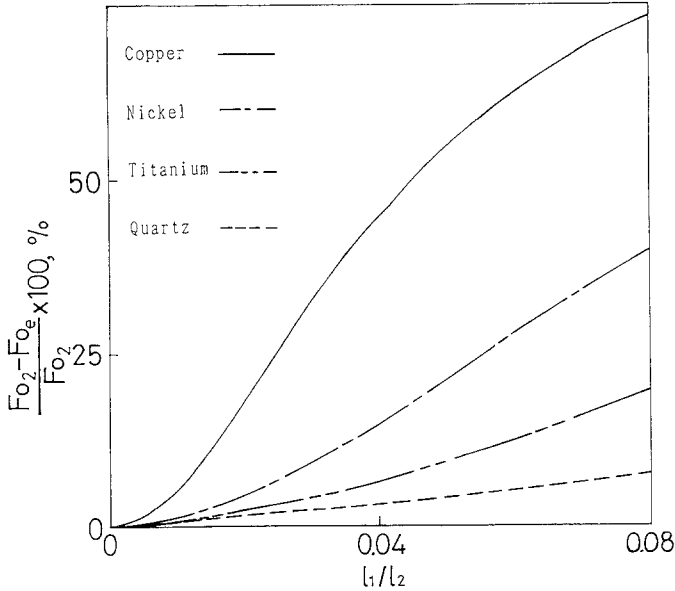


Fig. 8. Errors in thermal diffusivity of four different materials caused by the thickness of the carbon coating layer.

5. APPARENT AND MEAN THERMAL DIFFUSIVITIES

5.1. Approximate Expression of Apparent Thermal Diffusivity

There are needs to apply the transient methods for the evaluation of the thermal diffusivities of layered materials which are going to be used as electronic materials and materials resistance to wear, corrosion, and heat. For the direct evaluation, a definite relation between the apparent and the mean thermal diffusivities is indispensable because the temperature response at the rear surface of a sample only gives us the apparent thermal diffusivity even though information related to the thermophysical properties of the individual layers is contained in it.

In order to obtain the relation between the apparent and the mean thermal diffusivities, we first obtain an approximate expression for the apparent thermal diffusivity shown in Figs. 4-7. For this purpose, the perturbation method is applied around $\eta_{1/2} = 1$ and $Fo_2 = 0.5551$. If we let

$$\varepsilon = \eta_{1/2} - 1 \tag{19}$$

then the characteristic solution can be expressed as

$$2\gamma = (1 + \varepsilon\gamma_F + \varepsilon^2\gamma_S) k\pi \quad (k = 1, 2, \dots) \tag{20}$$

because the k th positive root is $2\gamma = k\pi$ for $\eta_{1/2} = 1$. Substituting Eq. (20) into the characteristic equation, Eq. (5), and comparing terms of the same order of ε , we obtain

$$\gamma_F = -\frac{1 + (2\kappa - 1)(-1)^k}{2}, \quad \text{and} \quad \gamma_S = \gamma_F^2 \quad (21)$$

where

$$\kappa = \frac{A_{1/2}}{1 + A_{1/2}} \quad (22)$$

Since we can express as

$$\omega_1 \chi_1 + \omega_2 \chi_2 = 2(1 + A_{1/2})(1 + \varepsilon\kappa) \quad (23)$$

$$\begin{aligned} &\omega_1 \chi_1 \cos(\gamma_k \omega_1) + \omega_2 \chi_2 \cos(\gamma_k \omega_2) \\ &= 2(1 + A_{1/2})(-1)^k \left[1 - \varepsilon\gamma_F - \varepsilon^2 \frac{1}{2} \left(k\pi \frac{1 + 2\gamma_F}{2} \right)^2 \right] \end{aligned} \quad (24)$$

the temperature rise in Eq. (1) for $V = 0.5$ becomes

$$0.5 = 1 + 2 \sum_{k=1}^{\infty} (-1)^k \frac{(1 + \varepsilon\kappa) \exp \left[-\left(\frac{k\pi}{2} \right)^2 (1 + \varepsilon\gamma_F + \varepsilon^2 \gamma_F^2)^2 \text{Fo}_2 \right]}{1 - \varepsilon\gamma_F - \varepsilon^2 \frac{1}{2} \left(k\pi \frac{1 + 2\gamma_F}{2} \right)^2} \quad (25)$$

If we further express

$$\text{Fo}_2 = \text{Fo}_{2,0}(1 + \varepsilon\mathcal{F}_F + \varepsilon^2\mathcal{F}_S); \quad \text{Fo}_{2,0} = 0.5551 \quad (26)$$

substitute Eq. (26) into Eq. (25), and compare the terms of the same order with respect to ε , we have the following relations.

$O(1)$:

$$\begin{aligned} -0.25 &= -x_0 + x_0^4 - x_0^9 + x_0^{16} - x_0^{25} + \dots; \\ x_0 &= \exp \left(-\frac{\pi^2}{4} \text{Fo}_{2,0} \right) = 0.25417 \end{aligned} \quad (27)$$

$O(\varepsilon)$:

$$\begin{aligned} 0 &= -x_0[(2\kappa - 1) + \{\mathcal{F}_F - 2(1 - \kappa)\} \ln(x_0)] + 4x_0^4(\mathcal{F}_F - 2\kappa) \ln(x_0) \\ &\quad - x_0^9[(2\kappa - 1) + 9\{\mathcal{F}_F - 2(1 - \kappa)\} \ln(x_0)] \\ &\quad + 16x_0^{16}(\mathcal{F}_F - 2\kappa) \ln(x_0) - \dots \end{aligned} \quad (28)$$

$O(\varepsilon^2)$:

$$\begin{aligned}
 0 = & -x_0 \left[\frac{(2\kappa - 1)\{\pi^2(2\kappa - 1) - 8(1 - \kappa)\}}{8} \right. \\
 & + \{\not{f}_S + \not{f}_F(4\kappa - 3) + (1 - \kappa)(5 - 7\kappa)\} \ln(x_0) \\
 & \left. + \frac{1}{2} \{\not{f}_F - 2(1 - \kappa)\}^2 \ln^2(x_0) \right] \\
 & + x_0^4 \left[\frac{4\pi^2(2\kappa - 1)^2}{8} + 4(\not{f}_S - 2\not{f}_F\kappa + 3\kappa^2) \ln(x_0) \right. \\
 & \left. + 8(\not{f}_F - 2\kappa)^2 \ln^2(x_0) \right] \\
 & - x_0^9 \left[\frac{(2\kappa - 1)\{9\pi^2(2\kappa - 1) - 8(1 - \kappa)\}}{8} \right. \\
 & + 9\{\not{f}_S + \not{f}_F(4\kappa - 3) + (1 - \kappa)(5 - 7\kappa)\} \ln(x_0) \\
 & \left. + \frac{81}{2} \{\not{f}_F - 2(1 - \kappa)\}^2 \ln^2(x_0) \right] \\
 & + x_0^{16} \left[\frac{16\pi^2(2\kappa - 1)^2}{8} + 16(\not{f}_S - 2\not{f}_F\kappa + 3\kappa^2) \ln(x_0) \right. \\
 & \left. + 128(\not{f}_F - 2\kappa)^2 \ln^2(x_0) \right] - \dots \tag{29}
 \end{aligned}$$

Evaluating Eqs. (28) and (29) with $x_0 = 0.25417$, we can derive the following equations:

$$\left(\frac{\not{f}_F}{2}\right) = 0.5 - 0.3592(\kappa - 0.5), \quad \frac{1}{2} \left[\not{f}_S - \left(\frac{\not{f}_F}{2}\right)^2 \right] = 0.3572(\kappa - 0.5)^2 \tag{30}$$

If we further note in Eq. (26) that

$$\text{Fo}_{2,0} = 4\text{Fo}_e = 4\eta_e^2 t_{1/2}; \quad \eta_e^2 = \frac{L^2}{a_e} \tag{31}$$

we obtain the approximate expression for the apparent thermal diffusion time as

$$\eta_e = 2\eta_1 [(A_F - 2A_S) + (1 - A_F + A_S) \eta_{2/1} + A_S \eta_{1/2}] \tag{32}$$

where

$$A_F = \frac{f_F}{2} = 0.5 - 0.3592(K - 0.5) \quad (33)$$

$$A_s = \frac{1}{2} \left[f_s - \left(\frac{f_F}{2} \right)^2 \right] = 0.3572(\kappa - 0.5)^2 \quad (34)$$

In Figs. 4-7, Eq. (32) is also shown by dashed curves. Although the present perturbation is conducted around $\eta_{1/2} = 1$, we see that the approximate expression of Eq. (32) is effective in the relatively wide, practically important range of $0.3 < \eta_{1/2} < 2$ even though $a_{1/2}$ varies from 1 to 10^3 . It is noted that Eq. (32) becomes an accurate expression when $A_{1/2} (= H_{1/2} \eta_{2/1}) = 1$.

5.2. Relation Between Apparent and Mean Thermal Diffusivities

Since we have derived the approximate expression for the apparent thermal diffusivity by Eq. (32), we can correlate it with the mean thermal diffusivity obtained from the thermal resistance. The mean thermal diffusivity a_m for the two-layered material is expressed as

$$\frac{L}{a_m \rho_m c_m} = \frac{l_1}{a_1 \rho_1 c_1} + \frac{l_2}{a_2 \rho_2 c_2} \quad (35)$$

with the relations of

$$\rho_m L = \rho_1 l_1 + \rho_2 l_2, \quad c_m \rho_m L = c_1 \rho_1 l_1 + c_2 \rho_2 l_2 \quad (36)$$

Since Eq. (35) can be rewritten as

$$\frac{L^2}{a_m} = \eta_m^2 = \eta_1^2 [(1 + H_{2/1}) + \eta_{2/1}^2 (1 + H_{1/2})] \quad (37)$$

we can express a definite relation between the apparent and the mean thermal diffusivities as follows:

$$\frac{a_m}{a_e} = \frac{4[(A_F - 2A_s) + (1 - A_F + A_s) \eta_{2/1} + A_s \eta_{1/2}]^2}{(1 + H_{2/1}) + \eta_{2/1}^2 (1 + H_{1/2})} \quad (38)$$

within the range where the approximate expression of the apparent thermal diffusivity can be applicable.

Once Eq. (38) has been derived, the mean thermal diffusivity can be obtained from the apparent thermal diffusivity when all the thermophysical

properties of individual layers are known. This condition may always be satisfied for the two-layered materials because we can anticipate that the thermal diffusivity of one layer and other individual thermophysical properties are known or easily determined and that the thermal diffusivity of the other layer is also determined by the method which has been used to determine thermal diffusivity of the material in the two-layered composite material. Then, after incorporating Eq. (38) into the measurement method with two-layered material described in Section 3, we can directly obtain the mean thermal diffusivity.

6. CONCLUDING REMARKS

In the present study, we have investigated the effects of various parameters on the measurement of the thermal diffusivity related to the transient method such as those with pulse- or stepwise heating. A two-layered sample material in the pulsewise heating method is considered because of its simplicity and usefulness in identifying and determining the effects of those parameters.

First, we have considered the situation to determine the thermal diffusivities of a material (two-layered sample) whose thermophysical properties except for the thermal diffusivity of one layer are known. Relations between parameters which affect the determination of the unknown thermal diffusivity have been clarified. It has been shown that there exists a special condition for the accurate measurement of thermal diffusivity. At this condition, the thermal diffusion time of each layer has the same value; accurate values of density and heat capacity of the material to be measured are not required. Then it is recommended to use this condition for the measurement of the thermal diffusivity for two-layered materials.

Second, we have considered the effect of laser-beam absorption layer which is often required for sample materials in the transient methods. Since these samples inevitably become two-layered materials, the thermal diffusivity obtained from the temperature response is merely the apparent one, which depends on the measurement method. Based on the examination of the dependence of the apparent thermal diffusivity of the two-layered material on the thermophysical properties and thickness of the layers, it has been shown that the thickness of the carbon coating layer may cause a relatively large error for materials of high thermal diffusivity.

Finally, we have examined the relation between the apparent thermal diffusivity obtained from the temperature response and the mean thermal diffusivity calculated from the thermal resistance. An approximate expression for the apparent thermal diffusivity has been analytically obtained in

the practically important range with the perturbation method, thereby a definite relation between the apparent and the mean thermal diffusivities is derived. After incorporating this relation into the measurement method with a two-layered material, the thermal conductivity as the heat-shield property of the two-layered material is anticipated to be determined directly.

ACKNOWLEDGMENT

This work was supported in part by the Promoting Foundation for Science and Technology on Functionally Gradient Materials, a contract funded through the National Aerospace Laboratory and sponsored by the Science and Technology Agency, Japan.

NOMENCLATURE

A	Variable to express the approximate expression of the apparent thermal diffusivity
a	Thermal diffusivity
c	Specific heat capacity
F_0	Fourier number [$= at/l^2$]
f	Perturbed value of the Fourier number
H	Heat capacity [$= c\rho l$]
l	Thickness
t	Time
V	Temperature ratio
γ	Positive root of the characteristic equation
ε	Small parameter defined by Eq. (19)
η	Thermal diffusion time [$= l/\sqrt{a}$]
κ	Parameter defined by Eq. (22)
λ	Thermal conductivity
ρ	Density
χ	Parameter defined in Eq. (2)
ω	Parameter defined in Eq. (2)

Subscripts

e	Apparent
i	Value of i th layer ($i = 1$ and 2)

- i/j Quantity of the i th layer divided by quantity of the j th layer ($i, j = 1, 2$)
 m Mean
F First-order approximation
S Second-order approximation

REFERENCES

1. T. Y. R. Lee, *Thermal Diffusivity of Dispersed and Layered Composites*, Ph.D. thesis (Purdue University, Lafayette, Ind. 1977).
2. N. Araki and K. Natsui, *Trans. Japan. Soc. Mech. Eng. (Ser. B)* **49**:1048 (1983) (in Japanese). [Translated into English, *Heat Transfer Japan. Res.* **14**:36 (1985).]
3. N. Araki, *Int. J. Thermophys.* **5**:53 (1984).
4. *JSME, JSME Data Book: Heat Transfer*, 4th ed. (JSME, Tokyo, 1986) (in Japanese).